

Exact Solutions of Bianchi Types I and V Models in $f(R, T)$ Gravity

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Abstract

This paper is devoted to investigate the exact solutions of Bianchi types I and V spacetimes in the context of $f(R, T)$ gravity [1]. For this purpose, we find two exact solutions in each case by using assumption of constant deceleration parameter and the variation law of Hubble parameter. The obtained solutions correspond to two different models of this universe. The physical behavior of these models is also discussed.

Keywords: $f(R, T)$ gravity, Bianchi types I and V , deceleration parameter.

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1 Introduction

The most popular issue in the modern day cosmology is the current expansion of universe. It is now evident from observational and theoretical facts that our

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universe is in the phase of accelerated expansion [2]-[10]. The phenomenon of dark energy and dark matter is another topic of discussion [11]-[18]. It was Einstein who first gave the concept of dark energy and introduced the small positive cosmological constant. But after that, he remarked it as the biggest mistake in his life. However, it is now thought that the cosmological constant may be a suitable candidate for dark energy. Another proposal to justify the current expansion of universe comes from modified or alternative theories of gravity. $f(T)$ theory of gravity is one such example which has been recently developed. This theory is a generalized version of teleparallel gravity in which Weitzenböck connection is used instead of Levi-Civita connection. The interesting feature of this theory is that it may explain the current acceleration without involving dark energy. A considerable amount of work has been done in this theory so far [19]. Another interesting modified theory is $f(R)$ theory of gravity. In this theory, a general function of Ricci scalar is used in standard Einstein-Hilbert lagrangian. Some review articles [20] can be helpful to understand the theory.

Many authors have investigated $f(R)$ gravity in different contexts. Spherically symmetric solutions are most commonly studied solutions due to their closeness to the nature. Multamäki and Vilja [21] explored vacuum and perfect fluid solutions of spherically symmetric spacetime in metric version of this theory. They used the assumption of constant scalar curvature and found that the solutions corresponded to the already existing solutions in general relativity (GR). Noether symmetries have been used by Capozziello et al. [22] to study spherically symmetric solutions in $f(R)$ gravity. Similarly many interesting results have been found using spherical symmetry in $f(R)$ gravity [23]. Cylindrically symmetric vacuum and non-vacuum solutions has also been explored in this theory [24]. Sharif and Shamir [25]. found plane symmetric solutions. The same authors [26] discussed the solutions of Bianchi types I and V cosmologies for vacuum and non-vacuum cases. We [27] calculated conserved quantities in $f(R)$ gravity via Noether symmetry approach.

In a recent paper [1], Harko et al. proposed a new generalized theory known as $f(R, T)$ gravity. In this theory, gravitational Lagrangian involves an arbitrary function of the scalar curvature R and the trace of the energy-momentum tensor T . Myrzakulov [28] discussed $f(R, T)$ gravity in which he explicitly presented point like Lagrangians. The exact solutions of $f(R, T)$ field equations for locally rotationally symmetric Bianchi type I spacetime has been reported by Adhav [29]. Sharif and Zubair [30] discussed the laws of

thermodynamics in this theory. Houndjo [31] reconstructed $f(R, T)$ gravity by taking $f(R, T) = f_1(R) + f_2(T)$ and it was proved that $f(R, T)$ gravity allowed transition of matter from dominated phase to an acceleration phase. Thus it is hoped that $f(R, T)$ gravity may explain the recent phase of cosmic acceleration of our universe. This theory can be used to explore many issues and may provide some satisfactory results.

In this paper, we are focussed to investigate the exact solutions of Bianchi types I and V spacetimes in the framework of $f(R, T)$ gravity. The plan of paper is as follows: In section 2, we give some basics of $f(R, T)$ gravity. Section 3 and 4 give the exact solutions for Bianchi types I and V spacetimes. Concluding remarks are given in the last section.

2 Some Basics of $f(R, T)$ Gravity

The $f(R, T)$ theory of gravity is the generalization or modification of GR. The action for this theory is given by [1]

$$S = \int \sqrt{-g} \left(\frac{1}{16\pi G} f(R, T) + L_m \right) d^4x, \quad (1)$$

where $f(R, T)$ is an arbitrary function of the Ricci scalar R and the trace T of energy momentum tensor $T_{\mu\nu}$ while L_m is the usual matter Lagrangian. It is worth mentioning that if we replace $f(R, T)$ with $f(R)$, we get the action for $f(R)$ gravity and replacement of $f(R, T)$ with R leads to the action of GR. The energy momentum tensor $T_{\mu\nu}$ is defined as [32]

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \quad (2)$$

Here we assume that the dependance of matter Lagrangian is merely on the metric tensor $g_{\mu\nu}$ rather than its derivatives. In this case, we obtain

$$T_{\mu\nu} = L_m g_{\mu\nu} - 2 \frac{\delta L_m}{\delta g^{\mu\nu}}. \quad (3)$$

The $f(R, T)$ gravity field equations are obtained by varying the action S Eq.(1) with respect to the metric tensor $g_{\mu\nu}$

$$f_R(R, T)R_{\mu\nu} - \frac{1}{2}f(R, T)g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square)f(R, T) = \kappa T_{\mu\nu} - f_T(R, T)(T_{\mu\nu} + \Theta_{\mu\nu}), \quad (4)$$

where ∇_μ denotes the covariant derivative and

$$\square \equiv \nabla^\mu \nabla_\mu, \quad f_R(R, T) = \frac{\partial f_R(R, T)}{\partial R}, \quad f_T(R, T) = \frac{\partial f_R(R, T)}{\partial T}, \quad \Theta_{\mu\nu} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}.$$

Contraction of (4) yields

$$f_R(R, T)R + 3\square f_R(R, T) - 2f(R, T) = \kappa T - f_T(R, T)(T + \Theta), \quad (5)$$

where $\Theta = \Theta_\mu{}^\mu$. This is an important equation because it provides a relationship between Ricci scalar R and the trace T of energy momentum tensor. Using matter Lagrangian L_m , the standard matter energy-momentum tensor is derived as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \quad (6)$$

where $u_\mu = \sqrt{g_{00}}(1, 0, 0, 0)$ is the four-velocity in co-moving coordinates and ρ and p denotes energy density and pressure of the fluid respectively. Perfect fluids problems involving energy density and pressure are not any easy task to deal with. Moreover, there does not exist any unique definition for matter Lagrangian. Thus we can assume the matter Lagrangian as $L_m = -p$ which gives

$$\Theta_{\mu\nu} = -pg_{\mu\nu} - 2T_{\mu\nu}, \quad (7)$$

and consequently the field equations (4) take the form

$$f_R(R, T)R_{\mu\nu} - \frac{1}{2}f(R, T)g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu}\square)f_R(R, T) = \kappa T_{\mu\nu} + f_T(R, T)(T_{\mu\nu} + pg_{\mu\nu}), \quad (8)$$

It is mentioned here that these field equations depend on the physical nature of matter field. Many theoretical models corresponding to different matter contributions for $f(R, T)$ gravity are possible. However, Harko et al. gave three classes of these models

$$f(R, T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R)f_3(T). \end{cases}$$

In this paper we are focussed to the first class, i.e. $f(R, T) = R + 2f(T)$. For this model the field equations become

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} + 2f'(T)T_{\mu\nu} + \left[f(T) + 2pf'(T)\right]g_{\mu\nu}, \quad (9)$$

where prime represents derivative with respect to T .

3 Exact Solutions of Bianchi Type *I* Universe

In this section, we shall find exact solutions of Bianchi I spacetime in $f(R, T)$ gravity. For this purpose, we use natural system of units ($G = c = 1$) and $f(T) = \lambda T$, where λ is an arbitrary constant. For Bianchi type *I* spacetime, the line element is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2, \quad (10)$$

where A , B and C are defined as cosmic scale factors. The Bianchi *I* Ricci scalar turns out to be

$$R = -2 \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} \right], \quad (11)$$

where dot denotes derivative with respect to t .

Using Eq.(9), we get four independent field equations,

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = (8\pi + 3\lambda)\rho - \lambda p, \quad (12)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \lambda\rho - (8\pi + 3\lambda)p, \quad (13)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{AC} = \lambda\rho - (8\pi + 3\lambda)p, \quad (14)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \lambda\rho - (8\pi + 3\lambda)p. \quad (15)$$

These are four non-linear differential equations with five unknowns namely A , B , C , ρ and p . Subtracting Eq.(14) from Eq.(13), Eq.(15) from Eq.(14) and Eq.(15) from Eq.(12), we get respectively

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = 0, \quad (16)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A} \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) = 0, \quad (17)$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{B}}{B} \left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C} \right) = 0. \quad (18)$$

These equations imply that

$$\frac{B}{A} = d_1 \exp \left[c_1 \int \frac{dt}{a^3} \right], \quad (19)$$

$$\frac{C}{B} = d_2 \exp \left[c_2 \int \frac{dt}{a^3} \right], \quad (20)$$

$$\frac{A}{C} = d_3 \exp \left[c_3 \int \frac{dt}{a^3} \right], \quad (21)$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are integration constants which satisfy the following relation

$$c_1 + c_2 + c_3 = 0, \quad d_1 d_2 d_3 = 1. \quad (22)$$

Using Eqs.(19)-(21), we can write the unknown metric functions in an explicit way

$$A = ap_1 \exp \left[q_1 \int \frac{dt}{a^3} \right], \quad (23)$$

$$B = ap_2 \exp \left[q_2 \int \frac{dt}{a^3} \right], \quad (24)$$

$$C = ap_3 \exp \left[q_3 \int \frac{dt}{a^3} \right], \quad (25)$$

where

$$p_1 = (d_1^{-2} d_2^{-1})^{\frac{1}{3}}, \quad p_2 = (d_1 d_2^{-1})^{\frac{1}{3}}, \quad p_3 = (d_1 d_2^2)^{\frac{1}{3}} \quad (26)$$

and

$$q_1 = -\frac{2c_1 + c_2}{3}, \quad q_2 = \frac{c_1 - c_2}{3}, \quad q_3 = \frac{c_1 + 2c_2}{3}. \quad (27)$$

It is mentioned here that p_1, p_2, p_3 and q_1, q_2, q_3 also satisfy the relation

$$p_1 p_2 p_3 = 1, \quad q_1 + q_2 + q_3 = 0. \quad (28)$$

3.1 Some Important Physical Parameters

Now we present some important definitions of physical parameters. The average scale factor a and volume scale factor V are defined as

$$a = \sqrt[3]{ABC}, \quad V = a^3 = ABC. \quad (29)$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \quad (30)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are defined as the directional Hubble parameters in the directions of x , y and z axis respectively. The mean anisotropy parameter A is

$$A = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2. \quad (31)$$

The expansion scalar θ and shear scalar σ^2 are defined as follows

$$\theta = u^\mu_{;\mu} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \quad (32)$$

$$\sigma^2 = \frac{1}{2} \sigma_{\mu\nu} \sigma^{\mu\nu} = \frac{1}{3} \left[\left(\frac{\dot{A}}{A} \right)^2 + \left(\frac{\dot{B}}{B} \right)^2 + \left(\frac{\dot{C}}{C} \right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA} \right], \quad (33)$$

where

$$\sigma_{\mu\nu} = \frac{1}{2} (u_{\mu;\alpha} h_\nu^\alpha + u_{\nu;\alpha} h_\mu^\alpha) - \frac{1}{3} \theta h_{\mu\nu}, \quad (34)$$

$h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ is the projection tensor.

The deceleration parameter q is the measure of the cosmic accelerated expansion of the universe. It is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \quad (35)$$

The behavior of the universe models is determined by the sign of q . The positive value of deceleration parameter suggests a decelerating model while the negative value indicates inflation. Since there are four equations and five unknowns, so we use a well-known relation [33] between the average scale factor a and average Hubble parameter H to solve the equations,

$$H = la^{-n}, \quad (36)$$

where l and n are positive constants.

Using Eqs.(30) and (36), we get

$$\dot{a} = la^{1-n} \quad (37)$$

and the deceleration parameter becomes

$$q = n - 1. \quad (38)$$

Integrating Eq.(37), it follows that

$$a = (nlt + k_1)^{\frac{1}{n}}, \quad n \neq 0 \quad (39)$$

and

$$a = k_2 \exp(lt), \quad n = 0, \quad (40)$$

where k_1 and k_2 are constants of integration. Thus we get two different models of the universe corresponding to these values of the average scale factor.

3.2 Singular Model of the Universe

Here we investigate the model of universe when $n \neq 0$, i.e., $a = (nlt + k_1)^{\frac{1}{n}}$. In this case, the metric coefficients A , B and C takes the form

$$A = p_1(nlt + k_1)^{\frac{1}{n}} \exp \left[\frac{q_1(nlt + k_1)^{\frac{n-3}{n}}}{l(n-3)} \right], \quad n \neq 3 \quad (41)$$

$$B = p_2(nlt + k_1)^{\frac{1}{n}} \exp \left[\frac{q_2(nlt + k_1)^{\frac{n-3}{n}}}{l(n-3)} \right], \quad n \neq 3 \quad (42)$$

$$C = p_3(nlt + k_1)^{\frac{1}{n}} \exp \left[\frac{q_3(nlt + k_1)^{\frac{n-3}{n}}}{l(n-3)} \right], \quad n \neq 3. \quad (43)$$

The directional Hubble parameters H_i ($i = 1, 2, 3$) turn out to be

$$H_i = \frac{l}{nlt + k_1} + \frac{q_i}{(nlt + k_1)^{\frac{3}{n}}}. \quad (44)$$

The mean generalized Hubble parameter and volume scale factor are

$$H = \frac{l}{nlt + k_1}, \quad V = (nlt + k_1)^{\frac{3}{n}}. \quad (45)$$

The mean anisotropy parameter becomes

$$A = \frac{q_1^2 + q_2^2 + q_3^2}{3l^2(nlt + k_1)^{(6-2n)/n}}. \quad (46)$$

The expansion scalar and shear scalar for this model are given by

$$\theta = \frac{3l}{nlt + k_1}, \quad \sigma^2 = \frac{q_1^2 + q_2^2 + q_3^2}{2(nlt + k_1)^{6/n}}. \quad (47)$$

Using Eqs. (12)-(15), the energy density of the universe is

$$\begin{aligned} \rho = & \frac{1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4(\lambda + 3\pi) \left\{ \frac{3l^2}{(nlt + k_1)^2} + \frac{q_1 q_2 + q_2 q_3 + q_3 q_1}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right. \\ & \left. - \lambda \left\{ \frac{3l^2(1 - n)}{(nlt + k_1)^2} + \frac{q_1^2 + q_2^2 + q_3^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right] \end{aligned} \quad (48)$$

while the pressure of the universe becomes

$$\begin{aligned} p = & \frac{-1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4\pi \left\{ \frac{3l^2}{(nlt + k_1)^2} + \frac{q_1 q_2 + q_2 q_3 + q_3 q_1}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right. \\ & \left. + (3\lambda + 8\pi) \left\{ \frac{3l^2(1 - n)}{(nlt + k_1)^2} + \frac{q_1^2 + q_2^2 + q_3^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right]. \end{aligned} \quad (49)$$

3.3 Non-singular Model of the Universe

For this model, $n = 0$ and the average scale factor $a = k_2 \exp(lt)$ turns the metric coefficients A , B and C into

$$A = p_1 k_2 \exp(lt) \exp \left[-\frac{q_1 \exp(-3lt)}{3lk_2^3} \right], \quad (50)$$

$$B = p_2 k_2 \exp(lt) \exp \left[-\frac{q_2 \exp(-3lt)}{3lk_2^3} \right], \quad (51)$$

$$C = p_3 k_2 \exp(lt) \exp \left[-\frac{q_3 \exp(-3lt)}{3lk_2^3} \right]. \quad (52)$$

The directional Hubble parameters H_i become

$$H_i = l + \frac{q_i}{k_2^3} \exp(-3lt). \quad (53)$$

The mean generalized Hubble parameter and volume scale factor turns out to be

$$H = l, \quad V = k_2^3 \exp(3lt). \quad (54)$$

The mean anisotropy parameter, expansion scalar and shear scalar are

$$A = \frac{q_1^2 + q_2^2 + q_3^2}{3l^2 k_2^6 \exp(6lt)}, \quad \theta = 3l, \quad \sigma^2 = \frac{q_1^2 + q_2^2 + q_3^2}{2k_2^6 \exp(6lt)}. \quad (55)$$

For this model, the energy density and pressure of the universe takes the form

$$\begin{aligned} \rho = & \frac{1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4(\lambda + 3\pi) \left\{ 3l^2 + \frac{q_1 q_2 + q_2 q_3 + q_3 q_1}{k_2^6 \exp(6lt)} \right\} \right. \\ & \left. - \lambda \left\{ 3l^2 + \frac{q_1^2 + q_2^2 + q_3^2}{k_2^6 \exp(6lt)} \right\} \right], \end{aligned} \quad (56)$$

$$\begin{aligned} p = & \frac{-1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4\pi \left\{ 3l^2 + \frac{q_1 q_2 + q_2 q_3 + q_3 q_1}{k_2^6 \exp(6lt)} \right\} \right. \\ & \left. + (3\lambda + 8\pi) \left\{ 3l^2 + \frac{q_1^2 + q_2^2 + q_3^2}{k_2^6 \exp(6lt)} \right\} \right]. \end{aligned} \quad (57)$$

4 Exact Bianchi Type V Solutions

Here we shall explore Bianchi type V solutions in the context of $f(R, T)$ gravity. The metric for the Bianchi type V spacetime is

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}[B^2(t)dy^2 + C^2(t)dz^2]. \quad (58)$$

Here A , B and C are also cosmic scale factors and m is an any constant. The Ricci scalar for this spacetime is

$$R = -2 \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{3m^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} \right]. \quad (59)$$

Using Eq.(9), we get

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda p, \quad (60)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)p, \quad (61)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{AC} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)p, \quad (62)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)p \quad (63)$$

and the 01-component turn out to be

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \quad (64)$$

We adopt the same procedure as for the Bianchi type *I* solutions to solve these equations. Here the equations Eqs.(16)-(18) are same as obtained previously but by making use of Eq.(64), we get the constraint equations as follows

$$p_1 = 1, \quad p_2 = p_3^{-1} = P, \quad q_1 = 0, \quad q_2 = -q_3 = Q. \quad (65)$$

Thus, the metric coefficients become

$$A = a, \quad B = aP \exp \left[Q \int \frac{dt}{a^3} \right], \quad C = aP^{-1} \exp \left[-Q \int \frac{dt}{a^3} \right]. \quad (66)$$

4.1 Singular Model of the Universe

For the model of the universe when $n \neq 0$, the metric functions A , B and C become

$$A = (nlt + k_1)^{\frac{1}{n}}, \quad (67)$$

$$B = P(nlt + k_1)^{\frac{1}{n}} \exp \left[\frac{Q(nlt + k_1)^{\frac{n-3}{n}}}{l(n-3)} \right], \quad n \neq 3 \quad (68)$$

$$C = P^{-1}(nlt + k_1)^{\frac{1}{n}} \exp \left[\frac{-Q(nlt + k_1)^{\frac{n-3}{n}}}{l(n-3)} \right], \quad n \neq 3. \quad (69)$$

The directional Hubble parameters H_1 , H_2 and H_3 take the form

$$H_1 = \frac{l}{nlt + k_1}, \quad (70)$$

$$H_2 = \frac{l}{nlt + k_1} + \frac{Q}{(nlt + k_1)^{\frac{3}{n}}}, \quad (71)$$

$$H_3 = \frac{l}{nlt + k_1} - \frac{Q}{(nlt + k_1)^{\frac{3}{n}}}. \quad (72)$$

The mean anisotropy parameter becomes

$$A = \frac{2Q^2}{3(nlt + k_1)^{(6-2n)/n}}. \quad (73)$$

The shear scalar for this model is given by

$$\sigma^2 = \frac{Q^2}{(nlt + k_1)^{6/n}}. \quad (74)$$

It is to be noticed that the mean generalized Hubble parameter H , expansion scalar θ and the volume scale factor V are same as in the case of Bianchi type I spacetime. The energy density and pressure of Bianchi V universe for this model turns out to be

$$\begin{aligned} \rho = & \frac{1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4(\lambda + 3\pi) \left\{ \frac{3l^2}{(nlt + k_1)^2} - \frac{Q^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right. \\ & \left. - \lambda \left\{ \frac{3l^2(1-n)}{(nlt + k_1)^2} + \frac{2Q^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right], \end{aligned} \quad (75)$$

$$\begin{aligned} p = & \frac{-1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4\pi \left\{ \frac{3l^2}{(nlt + k_1)^2} - \frac{Q^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right. \\ & \left. + (3\lambda + 8\pi) \left\{ \frac{3l^2(1-n)}{(nlt + k_1)^2} + \frac{2Q^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right]. \end{aligned} \quad (76)$$

4.2 Non-singular Model of the Universe

For the model when $n = 0$, the metric coefficients A , B and C turn out to be

$$A = k_2 \exp(lt), \quad (77)$$

$$B = Pk_2 \exp(lt) \exp \left[-\frac{Q \exp(-3lt)}{3lk_2^3} \right], \quad (78)$$

$$C = P^{-1}k_2 \exp(lt) \exp \left[\frac{Q \exp(-3lt)}{3lk_2^3} \right]. \quad (79)$$

The directional Hubble parameters H_1 , H_2 and H_3 are

$$H_1 = l, \quad H_2 = l + \frac{Q \exp(-3lt)}{k_2^3}, \quad H_3 = l - \frac{Q \exp(-3lt)}{k_2^3}. \quad (80)$$

The mean anisotropy parameter and shear scalar for this model become

$$A = \frac{2Q^2}{3l^2 k_2^6 \exp(6lt)}, \quad \sigma^2 = \frac{Q^2}{k_2^6 \exp(6lt)}. \quad (81)$$

Here we also get the same volume scale factor V , expansion scalar θ and mean generalized Hubble parameter H as shown in Eqs.(53)-(54). The energy density and pressure of universe here become

$$\rho = \frac{1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4(\lambda + 3\pi) \left\{ 3l^2 - \frac{Q^2}{k_2^6 \exp(6lt)} \right\} - \lambda \left\{ 3l^2 + \frac{2Q^2}{k_2^6 \exp(6lt)} \right\} \right], \quad (82)$$

$$p = \frac{-1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4\pi \left\{ 3l^2 - \frac{Q^2}{k_2^6 \exp(6lt)} \right\} + (3\lambda + 8\pi) \left\{ 3l^2 + \frac{2Q^2}{k_2^6 \exp(6lt)} \right\} \right]. \quad (83)$$

5 Concluding Remarks

This paper is devoted to discuss the current phenomenon of accelerated expansion of universe in the framework of newly proposed $f(R, T)$ theory of gravity. For this purpose, we take $f(R, T) = R + 2\lambda T$ and explore the exact solutions of Bianchi types I and V cosmological models. We obtain two exact solutions for both spacetimes using the assumption of constant value of deceleration parameter and the law of variation of Hubble parameter. The obtained solutions correspond to two different models of universe. The first solution forms a singular model with power law expansion while the second solution gives a non-singular model with exponential expansion of universe. The physical parameters for both of these models are discussed below.

The singular model of the universe corresponds to $n \neq 0$ with average scale factor $a = (nlt + k_1)^{\frac{1}{n}}$. This model possesses a point singularity when $t \equiv t_s = -\frac{k_1}{nl}$. The volume scale factor vanishes and the metric coefficients A , B and C vanish at this singularity point. The cosmological parameters H_1 , H_2 , H_3 , H , θ , and σ^2 are all infinite at this point of singularity. The mean anisotropy parameter A also becomes infinite at this point for $0 < n < 3$ and vanishes for $n > 3$. The energy density and pressure of universe are also infinite at this epoch. Moreover, the isotropy condition, i.e., $\frac{\sigma^2}{\theta} \rightarrow 0$ as $t \rightarrow \infty$, is verified for this model. All these conclusive observations suggest that the universe starts its expansion with zero volume, strong pressure and energy density from $t = t_s$ and it will continue to expand for $0 < n < 3$.

Now we discuss the non-singular model of the universe corresponds to $n = 0$. For this model the average scale factor is $a = k_2 \exp(lt)$. The non-singularity is due to the exponential behavior of the model. The expansion scalar θ and mean generalized Hubble parameter H are constant in this case. For finite values of t , the physical parameters H_1 , H_2 , H_3 , σ^2 and A are all finite. The metric functions are defined for finite time and the isotropy condition is satisfied. The pressure and energy density of universe become infinite in the limiting case when $t \rightarrow -\infty$. This shows that the universe evolved from an infinite past with a massive energy density and pressure. There is an exponential increase in the volume as the time grows. This shows that the universe started its expansion a long time ago with zero volume.

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